ON A METHOD FOR STUDYING TRANSIENTS IN NONLINEAR SYSTEMS

Abstract. The article considers transient processes in physical systems accompanied by vibrations.

The research carried out by the authors showed that when periodic external influences are applied, various cases are possible, and a sinusoidal external influence causes non-sinusoidal forced oscillation in a nonlinear system. The article shows that by introducing various nonlinearities, it is possible to change the properties of the system in a variety of ways, giving them new qualities that cannot be achieved in linear systems.

Keywords. Nonlinear system, transient process, properties, external influences, analysis, recommendations.

Introduction.

The need to understand and know the properties of nonlinear systems in railway transport is associated with the recently aggravated problem of train safety. Of course, it is very multifaceted, but one of its important issues is the problem of the behavior of the rail carriage as a nonlinear self-oscillating system. It is precisely this that represents the rail carriage in the horizontal plane of symmetry in the presence of random, and often unpredictable, external influences. There can be no doubt that this model is nonlinear, since the forces acting in contact between the wheel and the rail are nonlinear in nature and are called creep forces in the dynamics of rolling stock.

Let's consider one important case of a sliding mode in transients of nonlinear systems. In order to implement an ideal sliding mode in nonlinear systems, certain conditions must be met. A real system is always different from an ideal mathematical model. In this case, even a slight non-fulfillment of the conditions for the existence of an ideal sliding mode leads to the fact that theoretically the zero amplitude and infinite frequency of vibrations of the switching device (characterizing the ideal sliding mode) become finite. Therefore, in real systems, instead of an ideal sliding mode, a transition process consisting of a monotonous component and a vibration component superimposed on it is most often observed. Such a process could be called a real sliding mode, as opposed to an ideal one. However, it is also possible not to introduce such a name, since in this process some subtle signs characteristic of the accepted strict concept of the term "sliding mode" may be violated. Therefore, this type of transition process can be attributed simply to the category of various processes with a vibrational component. But since transients with vibrations can have a very different nature, let's agree that monotonous processes with vibrations in real systems, resulting from violation of the conditions of an ideal sliding regime, are called transients of a sliding type.
Materials and Methods.

In general, transients in physical systems accompanied by vibrations are characteristic of all self-oscillating nonlinear systems. If a control process is taking place in the system (so far we are talking about a transitional process), slowly changing over time compared to self-oscillations, the latter will be superimposed on the control process $x^0(t)$ in the form of a vibration component $x^*$ (Figure 1, a). Together with the change in the main component of the transition process $x^0(t)$ the amplitude will also change $A$ the vibration component $x^*(t)$, since in this case, the vibrations will capture various sections of the nonlinear characteristic (Figure 1, б). Here there is an interdependence of the two components of the process, which illustrates the illegality of the principle of superposition of solutions for nonlinear systems.

Even more diverse features are observed in the behavior of nonlinear systems in the presence of external influences (controlling and disturbing). First of all, we note that it was mentioned above about the free behavior of nonlinear systems with various initial deviations. Therefore, all of the above applies, in particular, to the behavior of the system under external influence in the form of an instantaneous impulse (push), the effect of which is to create the initial conditions of the process.

In the case of the application of a constant magnitude of impact (or for astatic systems – a constant rate of change of external influence), the system, after a transient process, enters a new steady-state mode in accordance with a nonlinear static characteristic. It is important to keep in mind that in self-oscillating systems characterized by a process $x(t) = x^0(t) + x^*(t)$ (Figure 1), static characteristic for the main signal $x^0(t)$ generally speaking, it will differ from the specified nonlinear characteristic. It is known, for example, that in a relay system in the presence of self-oscillating vibrations $x^*$ static characteristic for the main signal $x^0$ it will look like a smooth curve, and in the initial part it is close to linear (the effect of vibration smoothing of nonlinearity using self-oscillation).

Thus, the presence of self-oscillations changes the result of the action of an external influence, and this latter, in turn, as we already know, affects the magnitude of the amplitude (generally speaking, and frequency) of self-oscillatory vibrations according to the displacement $x^0$ by non-linear characteristic (Figure 1). Therefore, neither one nor the other component ($x^0$ и $x^*$) they cannot be determined in a nonlinear system independently of each other (the absence of the principle of superposition of solutions).

It is also interesting that the asymmetry of the nonlinear characteristic, which can itself create a static deviation (Figure 2), thus, it changes the entire steady-state mode. As a result, there is zero deviation $x^0$ it will not coincide with the zero value of the external influence.

As mentioned earlier, in the same nonlinear system, there may be two or more steady-state modes with corresponding regions of attraction according to initial conditions. In such a system, different values of constant external influence can cause fundamentally different behavior of the system. At the same time, the speed of application of external influence is also important. For example, its application by a leap can cause a cast ("overshoot") in the transient process, which will bring the system into a completely different area of attraction than with a smooth application of the same amount of external influence. In general, the behavior of the system can fundamentally change depending on the size of the external impact (a stable system can become self-oscillating or unstable, and vice versa).

Further, if an external influence is applied to the self-oscillating system, not constant, but slowly changing (compared with self-oscillations), then the overall picture of the process will be similar to the one described above, but with a slow change $x^0(t)$ and a slow change in the amplitude.
and frequency of the vibration component $x^*$. In the same way, each of them cannot be determined independently of the other.

Since the natural frequency of a nonlinear system can change significantly with a change in the amplitude of vibrations, resonant phenomena in nonlinear systems have a peculiar character. In the Figure 3 an example of a resonant curve of a nonlinear system is shown (dependence of the amplitude of forced oscillations $A_\theta$ from the frequency $\omega_\theta$ often referred to as the amplitude-frequency response: frequency response). At a given frequency $\omega_\theta$ the system may have more than one possible value of the amplitude of forced oscillations. Therefore, when the frequency changes $\omega_\theta$ amplitude jumps may occur.

Figure 1 - The control process in a nonlinear self-oscillating system

Figure 2 - Dependence of the threshold value of the amplitude of the external excitation acting on a nonlinear system on the frequency of the disturbance

Figure 3 - Amplitude-frequency response of a nonlinear system

Figure 4 - Partitioning of the range of parameters of the amplitude of an external disturbance $B$ and some parameter $k$, from the frequency of the disturbance
Self-oscillating systems, when applied to an external periodic effect, can perform complex movements as a result of the nonlinear superposition of their own and forced periodic oscillations. But with an increase in the amplitude $B$ of the external influence, a breakdown of self-oscillations may occur, and the system will completely switch to forced oscillations with a frequency $\omega_0$ external influence (synchronous mode). There is, therefore, a certain threshold value of the external amplitude $B$.

An example of the dependence of this value on the frequency is shown in Fig. 3.

Above this curve there will be a synchronous oscillation mode (with one frequency $\omega_0$). Threshold value of the external amplitude $B$ vanishes if there is a match $\omega_0$ with self-oscillation frequency $\omega_a$ of this system.

**Results and Discussion.**

Important features are obtained by control processes in the presence of external periodic effects (vibration interference or specially created external vibrations). The slope of the static characteristic of the nonlinear link (equivalent gain factor) for the passage of a slowly changing control process depends on the amplitude of the external vibration effect. Therefore, all the qualities of the control process can significantly depend on the amplitude of external vibrations in a nonlinear system, up to the occurrence of instability (in linear systems, as is known, their stability does not depend on external influences). This is related to the concept of vibration noise immunity of a nonlinear system, when the limit value of the amplitude of external vibration interference is determined, to which the system still remains stable. Figure 4 shows an example of the stability region by parameter $k$ and the external amplitude $B$. Such phenomena are inherent in many complex objects of transport equipment.

In other cases, external vibrations are used, on the contrary, to improve the properties of a nonlinear system. Thus, in systems with discontinuous and stagnant nonlinear characteristics (relays, dry friction, gap, dead zone), under the influence of external vibrations, the static characteristic of a nonlinear link for a slowly changing control signal becomes a smooth curve, close at its beginning to a straight line. This phenomenon is called vibrational smoothing of nonlinearity using forced vibrations (or vibrational linearization). According to this principle, a number of control and actuating devices of control (regulation) systems operate.

Finally, when external influences are random in nature, all phenomena and features of control processes similar to the above can occur in nonlinear systems, but they are described by methods of probability theory as random dynamic processes. At the same time, due to the nonlinearity of the system, all the statistical properties of these random processes are greatly complicated.

**Conclusions.**

In conclusion, we point out that by introducing various nonlinearities, it is possible to change the properties of the system in a variety of ways, giving them new qualities that cannot be achieved in linear systems. Therefore, nonlinear control laws and nonlinear correction devices, as well as systems with variable structure and logical control in general, are becoming increasingly widespread.

With the help of fairly simple non-linearities, it is often possible to achieve the same effects of improving the quality of the control process that are obtained in more complex self-adjusting systems. Therefore, nonlinear methods and devices are often used in self-tuning, self-learning and other types of adaptive systems.
When optimizing control systems according to various criteria, they most often come to nonlinear control laws with logical switches. The use of computer technology greatly expands the possibilities of implementation, as well as calculations in the design of any kind of nonlinear systems.

An analyst of a nonlinear system should use all the possibilities at his disposal to predict the behavior of the system. Often, he must be guided by intuition based on a fairly clear understanding of the physical behavior of the system, and not depend on the blind application of purely mathematical formulas. It is trivial to point out that such a representation helps to predict the solution in advance. Experimental information about the operation of the system can be of great importance in the study of its capabilities.

REFERENCES


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Received on June 19, 2023; Received in corrected form on July 11, 2023; Accepted on July 25, 2023.