DEVELOPMENT OF A MATHEMATICAL MODEL OF A CURRENT CONVERTER WITH NONLINEAR MAGNETIC RESISTANCE

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Abstract. A mathematical model of a current converter with nonlinear magnetic resistance has been developed, which makes it possible to increase the accuracy of the analytical calculation of magnetic circuits, taking into account the relationship between the specific magnetic resistance and magnetic induction. Methods for solving differential equations of magnetic circuits of a current converter having longitudinally distributed parameters are applied. An expression is obtained for the dependence of the specific magnetic resistance on magnetic induction.

Keywords: mathematical model, current converter, magnetic circuit, nonlinear magnetic resistance, magnetic flux, magnetomotive force, quasilinearization method, machine calculation method.

Introduction. Any magnetic circuits have a non-linearity of the magnetization curve. This is especially true for magnetic circuits of converters, the principle of operation of which is based on the use of the nonlinearity of the dependence of the specific magnetic resistance on magnetic induction. Therefore, to improve the accuracy of analytical calculations of the magnetic circuits of various converters, it is necessary to take into account the nonlinearity of the specific magnetic resistance of the magnetic materials from which they are made.

Materials and Methods. In the literature [1, 2], a number of methods for solving differential equations for magnetic circuits are proposed, taking into account the nonlinearity of the dependence of the specific magnetic resistance $\rho_{\mu}$ on the magnetic induction $B$ magnetic material. The approximation of the dependence $\rho_{\mu} = f(B)$ is carried out according to certain sections of the induction change, which greatly facilitates the solution of differential equations for a magnetic circuit with a single flow.

In the literature [3], the calculation of magnetic circuits is given taking into account the nonlinearity of the magnetization curve using a substitution scheme. The disadvantages of this method are, firstly, the use of approximation of the dependence $\rho_{\mu} = f(B)$ by a polynomial of the third degree, and secondly, the impossibility of determining the magnitude of the magnetic flux in any section of the magnetic circuit. Also in the work [3] the method of analytical calculation
of characteristics of magnetic material is given. This method is simple, but it is approximate, because when solving differential equations, an approximation of the dependence $\rho\mu = f(B)$ of the form is used

$$\rho\mu = \rho_{\mu0} + qB^2,$$

where, $\rho_{\mu0}$ – initial value of the specific magnetic resistance $[Gn^{-1}m^{-1}]$; $q$ – constant approximation coefficient $[Gn^{-1}m^{-1}Tl^{-2}]$.

**Results and Discussion.**

Known methods for calculating magnetic circuits give partial solutions and cannot be used for calculating magnetic circuits when high accuracy is required with a large range of changes in magnetic induction: $B = 0 + B_{\text{max}}$, as well as for magnetomodulation magnetic circuits. Therefore, it is necessary to develop more advanced methods for calculating magnetic circuits, taking into account the nonlinearity of the dependence of the specific magnetic resistance on the induction of magnetic material.

On the basis of the above calculation methods, the authors of this article have obtained a more accurate solution of differential equations for magnetic circuits: the laws of distribution of magnetic fluxes and magnetomotive force (MMF) along the length of the magnetic circuit are determined, taking into account the nonlinearity of the characteristics of the magnetic material of the magnetic circuit of a high-current electromagnetic converter (HCEC), the magnetic system of which is shown in the Figure 1 [4-6].

The converter consists of a magnetic circuit made of two (Fig.1) with – shaped parallel sections 1 and 2, interconnected by ferromagnetic rods of connecting elements 3 and 4, with rectangular cutouts, the parallelism of the connecting elements is maintained by means of wedges made of insulating material 5 and 6, a conductive bus 7, a winding 8 that creates a modulating magnetic flux $\Phi_m$, located uniformly on each rod of the connecting element of the measuring winding 9, simultaneously covering both rods of each connecting element and located along the edges of its cutout.

If a measured direct current $I_x$ is passed through the conductive bus 7, then counter-directed magnetic flows and are created in the left and right C–shaped magnetic conductors, which have maximum values in sections $A – A$ and $B – B$ (output windings 9 and 10 are located on the same sections, these windings are connected to each other in series, so the position of the output winding 10 in Fig.1b is not indicated). $\Phi_l$ and $\Phi_r$ by the middle of the connecting elements 3 and 4 in the section $O – O$ – decrease due to leakage of the magnetic flux through the air the gap.
Figure 1 - A magnetic system with a conductive bus (a) and the location of the modulating and output windings on the ferromagnetic connecting element (b) of the HCEC

The method of calculating such a magnetic circuit can be used to calculate magnetic circuits having \( \Pi \), \( O \), \( III \), \( S \)-shaped shapes.

For the HCEC magnetic circuit design shown in the figure, the cross sections of parallel magnetic conductors and the air gaps between them were considered constant, and buckling flows at the end and near the concentrated winding were not taken into account, since these issues were sufficiently fully investigated in [7], because these flows are very small compared to the main working magnetic flux. To determine the value of the controlled magnetic flux, taking into account the nonlinearity of the dependence of the specific magnetic resistance \( \rho_\mu = f(B) \) on induction, it is sufficient to consider second-order differential equations, which are the basic equations for magnetic circuits of the HCEC developed on the basis of longitudinally distributed parameters, which have the form:

\[
\phi_{1x}^* = 2gr_\mu \phi_{1x} = 2g \frac{\rho_\mu}{S_{st}} \phi_{1x},
\]

где \( S_{st} \) — section of the steel part of the connecting element \([m^2]\); \( g \) — magnetic conductivity \([Gn]\); \( r_\mu \) — linear magnetic resistance \([Gn^{-1}m^{-1}]\); \( \phi_{1x} \) — magnetic flux in the
steel part of the connecting element from the left section of the C-shaped magnetic circuit \([Vb]\); \(\Phi_{2x}\) – magnetic flux in the steel part of the connecting element from the right section of the C-shaped magnetic circuit \([Vb]\).

\[
\Phi_{2x} = 2g \rho_{\mu} \Phi_{2x} = 2g \frac{\rho_{\mu}}{S_{st}} \Phi_{2x}; \tag{2}
\]

If we replace \(\rho_{\mu}\) it with an approximating expression \(\rho_{\mu 0} + KB^6\), then equations (1) and (2) take the form

\[
\Phi_{1x} = 2gr_{\mu}S_{st}^{-1}(\rho_{\mu 0} + KB^6)\phi_{1x}; \tag{3}
\]

\(K\) is the approximation coefficient \([Gn^{-1}m^{-1}Tl^{-6}]\); \(B\) – magnetic induction \([Tl]\).

\[
\Phi_{2x} = 2gr_{\mu}S_{st}^{-1}(\rho_{\mu 0} + KB^6)\phi_{2x}; \tag{4}
\]

after the conversion, we get

\[
\Phi_{1x} = A_0 \phi_{1x} + A_{01} \phi_{1x}, \tag{5}
\]

\[
\Phi_{2x} = A_0 \phi_{2x} + A_{01} \phi_{2x}. \tag{6}
\]

where \(A_0 = 2gS_{st}^{-1} \rho_{\mu} \left[ m^2 \right]; A_{01} = 2gK \frac{1}{S_{st}} \left[ Gn^{-1}m^{-2} \right]\), to solve equations (5) and (6), it is assumed \(\rho_{\mu} \approx \rho_{\mu \text{av}}\) that the value \(\rho_{\mu \text{av}}\) - the average value is determined by the formula [1].

\[
\rho_{\mu \text{av}} = 0.5(\rho_{\mu 0} + \rho_{\mu \text{max}}) = \rho_{\mu 0} + 0.5KB^6. \tag{7}
\]

where \(\rho_{\mu 0}\) and \(\rho_{\mu \text{max}}\) – the initial and maximum value of the specific magnetic resistance.

Equations (5) and (6) are solved by the classical method recommended in [2].

Analysis of the solution of equations (5) and (6) showed that the dependence of the calculated magnetic flux on the cross-section coordinate \(A - A\), \(\Phi_{\text{wp}} = f_1(X)\), determined at \(\rho_{\mu} = \rho_{\mu \text{av}}\) differs from the dependence \(\Phi_{\text{wp}} = f(X)\), obtained experimentally by 8...10%, and the dependence of the calculated magnetic flux on the cross-section coordinate \(B - B\), \(\Phi_{\text{wp}} = f_2(X)\), determined at \(\rho_{\mu} = \rho_{\mu \text{av}}\) differs from the experimentally obtained by 12...15%.

Equations (5) and (6) of the magnetic flux, defined when \(\rho_{\mu} = \rho_{\mu \text{av}}\) cumbersome and time-consuming in the calculation. This limits the use of such a method when calculating a magnetic circuit, taking into account the nonlinearity of the dependence \(\rho_{\mu} = f(B)\). The numerical
analysis method is more suitable for calculating the magnetic circuit of the HCEC, taking into account the nonlinearity \( \rho_\mu = f(B) \).

The most promising method for solving a nonlinear boundary value problem is the quasi-linearization method, which allows reducing a nonlinear problem to a sequence of linear problems [8].

Let’s write down a boundary value problem for a magnetic circuit taking into account the nonlinearity \( \rho_\mu = f(B) \):

\[
\Phi' = 2r_\mu g\Phi; \quad \Phi = \begin{cases} 
\Phi_1 & \text{by } 0 \leq x \leq x_n \\
\Phi_2 & \text{by } x_n \leq x \leq x_M
\end{cases}
\]  

(8)

where \( x \) – rod length \([m]\); \( x_n \) – the cross-section coordinate \( O-O [m] \); \( x_M \) – the cross-section coordinate \( B-B [m] \).

With boundary conditions:

\[
\Phi|_{x=0} - \frac{1}{gZ_{\mu 0}} \Phi'|_{x=0} = 0;
\]

(9)

where \( Z_{\mu 0} \) – magnetic resistance of the steel part of the converter \([Gn^{-1}]\).

\[
\Phi|_{x=x_M} - \frac{1}{gZ_{\mu 0}} \Phi'|_{x=x_M} = 0;
\]

(10)

where \( Z_{\mu 0} \) – magnetic resistance of the air gap \([Gn^{-1}]\).

\[
\Phi|_{x=x_M} = \Phi|_{x=x_N};
\]

(11)

\[
\Phi'|_{x=x_M} - \Phi'|_{x=x_N} = gF_B.
\]

(12)

where \( F_B \) – magnetic voltage \([A]\).

Since (8) is a strictly convex function, i.e. \( \Phi'' > 0 \), to solve the problem (8) ... (12), can be used the quasilinearization method. For the initial approximation, we will take the solution of a linear problem, for this we take the value of the magnetic resistance \( r_\mu \) constant. Consider the sequence \( \{\Phi_n\} \), defined by the recurrence relation

\[
\Phi_n = f(\Phi_{n-1}) + (\Phi_n - \Phi_{n-1})f'(\Phi_{n-1}).
\]

(13)

let’s introduce the notation \( f_{n-1} = f'(\Phi_{n-1}) \), \( \psi_{n-1}(x) = f(\Phi_{n-1}) + f'(\Phi_{n-1})\Phi_{n-1} \).

Then the ratio (13) will take the form

\[
\Phi_n = f_{n-1}(\Phi_{n-1}) + \psi_{n-1}.
\]

(14)
We approximate (14), (8), (12) on the grid

$$\mathbf{W} = \left\{ x_n = \kappa H, \kappa = 0, \ldots, M, H = \frac{x_n}{M} \right\}$$,

where $H$ – grid step by $\kappa$; $M$ – the number of nodes in the grid by $\kappa$; $\kappa$ – number of steps by $\kappa$. We get

$$\begin{equation}
\begin{aligned}
\frac{\Phi_n \varphi_x - 2 H (\Phi_{n-1} - g Z_{\mu} \Phi_{n0})}{H} x = 0 \\
\frac{2 H (\Phi_{n-1} - g Z_{\mu} \Phi_{n0})}{H} x = x_m \\
-f_{n-1} (x_n) \Phi_{n_1} = \psi_{n-1} (x_k)
\end{aligned}
\end{equation}$$

Given (8), we introduce the notation

$$f_{n-1} (x_k) = \begin{cases} f_{i(n-1)} (x_k), & 0 \leq x \leq x_n; \\
f_{2(n-1)} (x_k), & x_n < x \leq x_m; \\
f_{2(n-1)} (x_k), & x_N \leq x \leq x_m; \end{cases}$$

$$\psi_{n-1} (x_k) = \begin{cases} \psi_{i(n-1)} (x_k), & 0 \leq x \leq x_n; \\
\psi_{2(n-1)} (x_k), & x_N \leq x \leq x_m; \end{cases}$$

then equation (15) in finite differences will be written as

$$A_k \Phi_{n(k-1)} - C_k \Phi_{n_k} + B_k \Phi_{n(k+1)} + F_k,$$

in equation (16)

$$A_k = \begin{cases} A_{1k}, & 0 \leq x \leq x_n; \\
A_{2k}, & x_n \leq x \leq x_m; \end{cases}$$

$$B_k = \begin{cases} B_{1k}, & 0 \leq x \leq x_n; \\
B_{2k}, & x_n \leq x \leq x_m; \end{cases}$$

$$F_k = \begin{cases} F_{1k}, & 0 \leq x \leq x_n; \\
F_{1k}, & x = x_n; \\
F_{1k}, & x_n \leq x \leq x_m. \end{cases}$$

At the same time

$$A_{1k} = A_{2k} = \frac{1}{H^2};$$

$$C_{1k} = \frac{2}{H^2} + f_{i(n-1)} (x_k);$$

$$B_{1k} = B_{2k} = \frac{1}{H^2};$$

$$F_k = \psi_{i(n-1)} (x_k);$$

for direct running:

$$C_{2k} = \frac{2}{H^2} + f_{2(n-1)} (x_k);$$
for the zero approximation: \[
\begin{aligned}
C_{1k} &= C_{2k} = \frac{2}{H^2} + 2r_\mu g, \\
F_{1k} &= F_{2k} = 0
\end{aligned}
\]

Where any of the known methods of solving systems of linear equations can be applied to solve (16). To solve the resulting system (13) and (14), the run-through method was applied, for the solution \( \Phi_1 \) – the left run, for the solution \( \Phi_2 \) - the right run.

We determine the run - through coefficients \( \alpha \) and \( \beta \) by the following formulas:

\[
\alpha_{k+1} = \frac{B_{1k}}{C_{1k} - \alpha_{k+1}A_{1k}}; \quad \beta_{k+1} = \frac{A_{1k}B_{2k} + F_{1k}}{C_{1k} - \alpha_{k+1}B_{2k}}; \quad k = 1, \ldots, n - 1; \quad \alpha_i = \frac{gZ_{\mu st}H}{gZ_{\mu st}H + 1}; \quad \beta_i = 0 .
\]

\[
\Phi_{1\beta k} = \alpha_{k+1}\Phi_{1\beta \{k+1\}}; \quad k = 0, \ldots, n - 1;
\]

where \( \beta_1 \) – the attenuation coefficient of the magnetic flux from the left section.

For the reverse course,

where \( \beta_2 \) – the attenuation coefficient of the magnetic flux from the right section.

\[
\alpha_k = \frac{A_{2k}}{C_{2k} - \alpha_{k+1}B_{2k}}; \quad \beta_k = \frac{B_{2k}B_{2k} + F_{2k}}{C_{2k} - \alpha_{k+1}B_{2k}}; \quad k = n + 1, \ldots, m - 1; \quad \alpha_m = \frac{gZ_{\mu st}H}{gZ_{\mu st}H + 1}; \quad \beta_m = 0 ;
\]

\[
\Phi_{2\beta (k+1)} = \alpha_{k+1}\Phi_{2\beta k} + \beta_{k+1}; \quad k = 0, \ldots, n - 1; \quad \Phi_{1\alpha n} = \Phi_{2\alpha n} = \frac{F_{1n}gH + \beta_n + \beta_{n+1}}{2 - \alpha_{n+1} - \alpha_n}.
\]

The boundary value problem (8) ... (12) is solved for two cases, we determine the linear magnetic resistance \( r_\mu \) using the approximation curve in the fifth and sixth degrees of magnetic induction, respectively, with the following data for the magnetic circuit in question:

\[
F_n = 100 \text{ [A]; } Z_{\mu st} = 1.1493 \cdot 10^6 \text{ [Gn\textsuperscript{-1}]; }
\]

\[
g = 0.447 \cdot 10^{-5} \text{ [Gn]; } x_m = 0.25 \text{ [m]; }
\]

\[
Z_{\mu 0} = 4.2576 \cdot 10^4 \text{ [Gn\textsuperscript{-1}]; } x = (0.01...0.25) \text{ [m].}
\]

Let’s consider two cases.

The first case is when \( r_\mu \) is determined using an approximation curve by a polynomial in the fifth degree of magnetic induction:

\[
r_\mu = \frac{r_\mu}{S} \leq \frac{1}{S_{\mu 0}} \left( \rho_{\mu 0} + KB^5 \right).
\]  

(17)

\( r_\mu \) – linear magnetic resistance [Gn\textsuperscript{-1}M\textsuperscript{-1}];
\[ \rho_\mu \] – specific magnetic resistance \([\text{Gn}^{-1}\text{m}^{-1}]\);

\(K\) - coefficient of approximation of the magnetization curve.

Substituting the values \(r_\mu\) from (17) to (8), we obtain, respectively, the following expressions for \(\Phi_1^*\) and \(\Phi_2^*\):

\[
\Phi_1^* = A_0 \frac{\Phi_1}{1 + A_1 \sinh(2A_1 \Phi_1)} + A_2 \Phi_1^6;
\]

\[
\Phi_2^* = A_0 \frac{\Phi_2}{1 + A_1 \sinh(2A_1 \Phi_1)} + A_3 \Phi_2^6.
\]

From (18) and (19) we determine the constant coefficients \(A_0, A_1, A_2, A_3\), for the magnetic circuit in question, they have the following values:

\[
A_0 = \frac{2\rho_\mu S_{cr}}{S_{cr}} = 1.83 \text{ [m}^{-2}\]; \quad A_1 = \kappa_1 = 10;
\]

\[
A_2 = \frac{K_2}{S_{st}} = 1.53 \cdot 10^4, \text{ [Gn}^{-1}\text{m}^{-2}\]; \quad A_3 = \frac{K_3}{S_{st}} = 15.15 \cdot 10^{20} \text{ [Gn}^{-1}\text{m}^{-2}\].
\]

Equations (18) and (19) will be written in general form according to [9-12]:

\[
\tilde{f}(\Phi) = \frac{A_0 \Phi}{1 + A_1 \sinh(A_2 \Phi)} + A_3 \Phi^6.
\]

With a zero approximation, we have the following expressions:

\[
f_{n-1}(\Phi) = \frac{A_0 \Phi_{n-1}}{1 + A_1 \sinh(A_2 \Phi_{n-1})} + A_3 \Phi_{n-1}^6;
\]

\[
\psi_{n-1}(x) = \tilde{f}_{n-1}(x) + \left[ A_0 + \frac{1}{2} A_1 A_2 \left(e^{\lambda_0 \Phi_{n-1}} - e^{-\lambda_0 \Phi_{n-1}}\right) - \frac{1}{2} A_0 A_1 A_2 \Phi_{n-1} \left(e^{\lambda_0 \Phi_{n-1}} + e^{-\lambda_0 \Phi_{n-1}}\right) + 6A_3 \Phi_{n-1}^5 \right] \Phi_{n-1}.
\]

The second case is when \(r_\mu\) is determined using the approximation curve by a polynomial in the sixth degree of magnetic induction:

\[
r_\mu = \rho_\mu \frac{S}{S_{st}} = \frac{1}{S_{st}} \left(\rho_{\mu 0} + KB \Phi_0^6\right).
\]

Substituting expressions for the value \(r_\mu\) from (23) to (8), we obtain, respectively, the following expressions for \(\Phi_1^*\) and \(\Phi_2^*\):
\[ \Phi_1^* = A_{01} \Phi_1 + A_{02} \Phi_1^7; \]
\[ \Phi_2^* = A_{01} \Phi_2 + A_{02} \Phi_2^7; \]  \hspace{1cm} (24)
\[ \Phi_3^* = A_{01} \Phi_3 + A_{02} \Phi_3^7; \]  \hspace{1cm} (25)

where, \( A_{01} = \frac{2}{S_{st}} g \rho_{\mu 0} = 11.17 \) [M^2]; \( A_{02} = \frac{2}{S_{st}} g K = 1755 \cdot 10^{20} \) [Gn^6 M^{-14}].

Equations (24) and (25) will be written in general form according to [9-12]:

\[ \tilde{f}(\Phi) = A_{01} \Phi + A_{02} \Phi^7 \]  \hspace{1cm} (26)

For the zero approximation we have:

\[ \tilde{f}_{n-1}(x) = A_{1} \Phi_{n-1} + A_{2} \Phi_{n-1}^7; \]  \hspace{1cm} (27)
\[ \psi_{n-1}(x) = \tilde{f}_{n-1}(x) \left( A_{01} + 7 A_{02} \Phi_{n-1}^6 \right) \Phi_{n-1}. \]  \hspace{1cm} (28)

The problems of differential equations of magnetic circuits of the HCEC, taking into account the nonlinearity \( \rho_{\mu} = f(B) \), were solved on a computer using the MatLab program. The results of solving nonlinear and linear problems coincided with an accuracy of up to the fourth digit.

**Conclusion.**

Studies have shown that the analytical method for calculating magnetic circuits at is \( \left( \rho_{\mu} = \text{const} \right) \) sufficient for calculating the magnetic circuits of the HCEC.

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